Considering Input and Output Impedances

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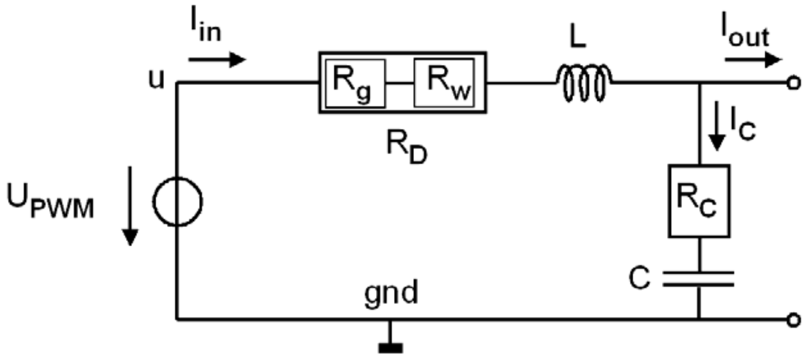
Abstract. This document illustrates how to cope with input and output (I/O) impedances.

1 Introduction

1.1 Motivation

It is important to consider I/O impedances of instrumentation. The *RLC* oscillator in Fig. 1 for example is typically computed as function of the capacitor's equivalent series resistor and the wire resistor of the coil, represented by R_C and R_w in Fig. 1, respectively, which are typically in the range of $\leq 0.1\Omega$. Consequently, the dominating resistor in this configuration is the output impedance of the signal source, which is typically $R_g = 50\Omega$ if a waveform generator is employed. In addition, monitor units may have impact on measured results.

Fig. 1: *RLC* oscillator



1.2 Outline

The organization of this communication is as follows:
Section 1 introduces into this document
Section 2 presents fundamental equations
Section 3 copes with output impedances and
Section 4 deals with input impedances, while
Section 5 concludes this document and
Section 6 offers some references

2 Fundamentals

The voltage divider in Fig. 2 delivers for output voltage U_o and source voltage U_s

$$\alpha = \frac{U_o}{U_s} = \frac{Z_2}{Z_1 + Z_2} \qquad (1.1)$$

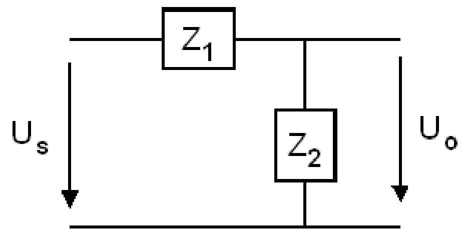


Fig. 2: voltage divider

and consequently

$$Z_1 = \frac{1-\alpha}{\alpha} Z_2 \qquad (1.2)$$

$$Z_2 = \frac{\alpha}{1-\alpha} Z_1 \qquad (1.3)$$

3 Measuring Output Impedances

3.1 DC Output Impedance

The voltage divider in Fig. 1 delivers for output voltage U_o and source voltage U_s

$$\alpha = \frac{U_o}{U_s} = \frac{R_L}{R_{out} + Z_L} \qquad (2.1)$$

and consequently

$$R_o = \frac{1-\alpha}{\alpha} R_L \qquad (2.2)$$

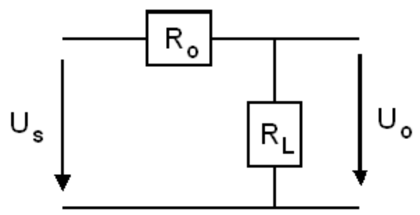


Fig. 3.1: voltage divider

Step 1: Measure U_s via $U_o = U_s$ using $R_L \rightarrow \infty$,
Step 2: Measure U_s with a load resistor R_L with similar size as R_o,
Step 3: Compute α according to (2.1),
Step 4: Compute R_o according to (2.1).

3.2 AC Output Impedance

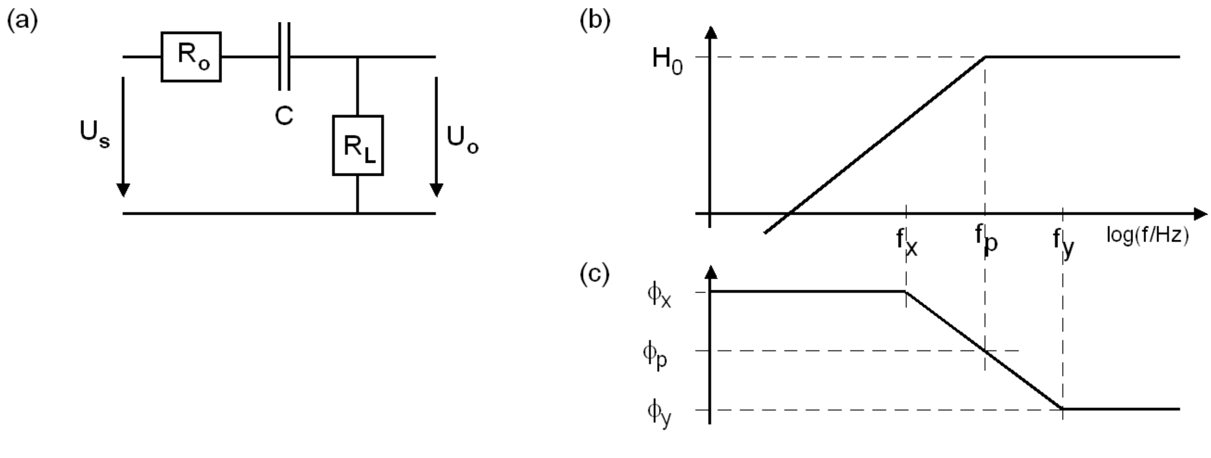


Fig. 3.2: (a) schematic, **(b)** amplitude- und **(c)** phase-diagramm.

Its transfer function is

$$H(s) = \frac{U_o(s)}{U_s(s)} = \frac{H_\infty}{1 + s/\omega_p} \qquad (2.3)$$

with

$$H_\infty = \frac{R_L}{R_o + R_L} \qquad (2.4)$$

and

$$\omega_p = \frac{1}{R_{oL}C} \quad \Leftrightarrow \quad f_p = \frac{1}{2\pi R_{oL}C} \qquad (2.5)$$

and

$$R_{oL} = R_o + R_L \qquad (2.6)$$

4 Measuring Input Impedances

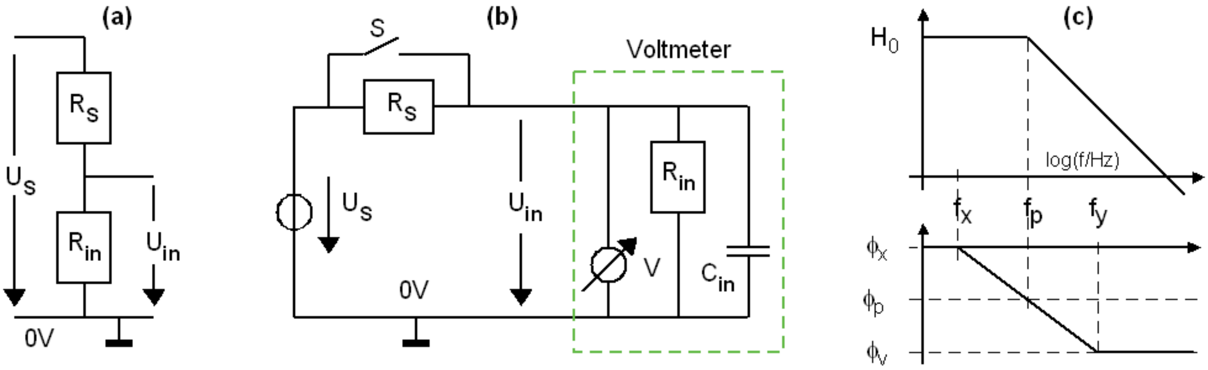


Fig. 4: (a) resistive voltage divider, **(b),** circuit to measure R_{in}, C_{in}, **(c)** Bode diagram

4.1 DC Input Impedance

The Transfer function of the circuit in Fig. 4(a) is

$$H_0 = \alpha = \frac{U_{in}}{U_S} = \frac{R_{in}}{R_S + R_{in}} \qquad (3.1)$$

with U_S and R_S being the source voltage and shunt resistor, respectively, and U_{in} input voltage of the voltmeter. U_S can be measured either with closed switch S or we simply measure both U_S and U_{in}, for example on the two channels of an oscilloscope. The input impedance of the voltmeter can be measured as

$$R_{in} = \frac{\alpha}{1-\alpha} R_S \qquad (3.1)$$

4.2 AC Input Impedance

The Transfer function of the circuit in Fig. 4(b) with open switch S is

$$H(s) = \frac{U_{in}}{U_S} = \frac{H_0}{1 + s/\omega_p} \qquad (3.2)$$

with

$$R_p = R_o \| R_L = \frac{R_o \cdot R_L}{R_o + R_L} \qquad (3.3)$$

and

$$\omega_p = \frac{1}{R_p C_{in}} \Leftrightarrow f_p = \frac{1}{2\pi R_p C_{in}} \qquad (3.4)$$

The input capacitor of the voltmeter, C_{in}, can then be computed from

$$C_{in} = \frac{1}{2\pi R_p f_p} \qquad (3.5)$$

Step 1: Measure Rin at $f = 0$ Hz or at least at $f << f_p$.
Step 2: Compute $H_0 = \alpha$ and R_p according to (3.1) and (3.3).
Step 3: Figure out f_p. At this frequency $H_0 = 1/\sqrt{2} = 0.707$ and the phase shift is 45°.

5 Conclusions

Some simple methods to compute input and output impedances of measurement equipment were presented.

6 References

[1] *OTH Regensburg*, available www.oth-regensburg.de