THE TRILOPEDE

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Abstract

The development of the Trilopede robot was inspired by a number of research projects dealing with the navigation of mobile robots over planar surfaces (solar cell cleaning, floor service robots, etc.). In all these cases, mobile robots are required to be capable of motion over planar terrain and to be able to reach into every corner of a given field of operation. This is not usually possible with conventional wheeled robots.

Key words: Trilopede, mobile robot, matrix kinematics

I. Introduction

Since its original phylogeny by Walch [Walch, 1771], the Trilobite serves as something of an anomaly in the history of extinct marine arthropods. In fact nature gave up on the idea of such three member arachnomorpha about 250 million years ago leaving bipedal (and integer multiples thereof) creatures to dominate the world. Nevertheless, for reasons of manoeuvrability, three wheeled mobile robots are fairly common [McKerrow, 1991] and some attempts in making three legged mobile robots have also been made [How & Amin, 2002]. However, hexapod and multiple joint legged robots are mechanically complicated and often pose problems to the development of a suitable kinematic model.

A more appropriate concept is that of a caterpillar, where motion is achieved by the forward extension of the frontal body parts while prehension of the substratum is maintained by the legs. Meanwhile, the rear part of the body is pulled forward by means of longitudinal dorsal and ventral muscles. This procedure is repeated to produce waves of forward locomotion [Blaney, 1977]. The simplest biomimetic realisation of this concept comprises two actuators and three feet in a linear chain configuration. The main problem with this design lies in the inherent singularity when both actuators are aligned along a common axis

II. Basic Concept

In order to avoid such singularities whilst minimising redundancy, the Trilopede comprises three linear actuators and three switchable feet. As shown in figure 1a and figure 1b, two basic configurations (delta and star respectively) are possible [Monkman et al, 2010].

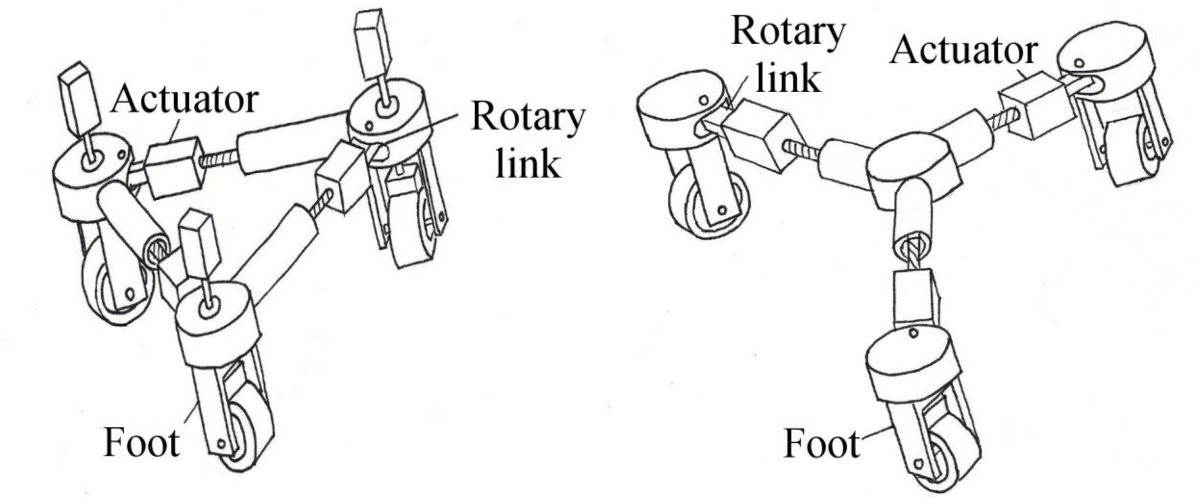

Fig 1: Trilopede configuration a) Delta b) Star

The actuators used may be simple pneumatic cylinders for discrete step motion, as shown in figure 2a, or electrically driven proportional actuators as shown in figure 2b. Hydraulic actuators are also conceivable but perhaps less practical for small scale devices.

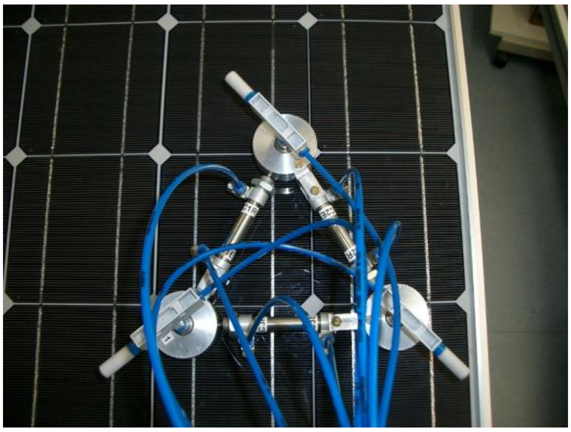
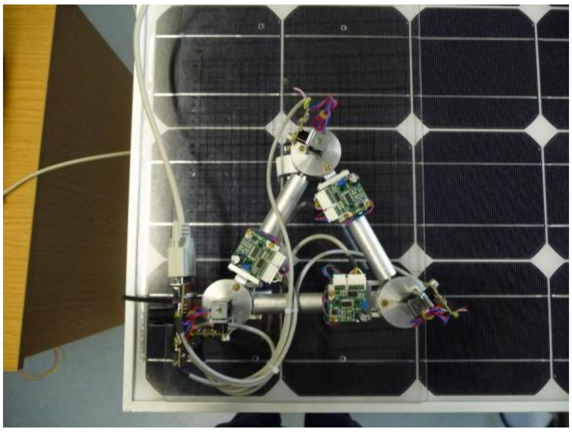

Fig 2: Trilopede actuation a) Discrete b) Proportional

Motion of the robot is achieved through a sequence of extracting and retracting the different actuators while the relative motion of two of the three feet is restricted in relation to the ground. The feet may be held in position by simple suction caps or more complicated mechanism [Wahler, 2010].

III. Kinematics

This section introduces a basic matrix based kinematic model for the Trilopede. The simplest mathematical representation of the Trilopede, independent of its physical realisation, is a triangle. In order to define a unique position of the Trilopede on a 2D-plane the coordinates of the centroid of the triangle together with the vector from the centroid to one or the vertices will be used as depicted in figure 3.

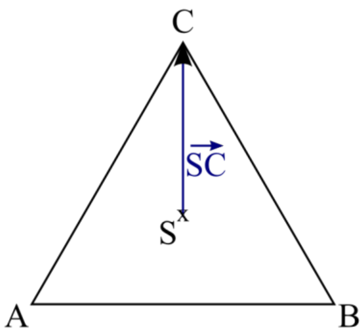

Fig 3: Mathematical abstraction of the trilopede

With these 4 coordinates (S_x, S_y, SC_x, SC_y) and the some basic geometry it is possible to calculate the coordinates of the three vertices. The coordinates of a point $P(P_x, P_y)$ can also be expressed as the vector $\overrightarrow{OP} = \begin{pmatrix} P_x \\ P_y \end{pmatrix}$ from the origin towards the point P. The coordinates of the vertices can be calculated as follows:

$$\overrightarrow{OA} = \overrightarrow{OS} + R(120°) \cdot \overrightarrow{SC} \qquad \{1\}$$

$$\overrightarrow{OB} = \overrightarrow{OS} + R(-120°) \cdot \overrightarrow{SC} \qquad \{2\}$$

$$\overrightarrow{OC} = \overrightarrow{OS} + \overrightarrow{SC} \qquad \{3\}$$

Where $R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ is the 2D-rotation matrix.

For the kinematics the following types of motion will be further discussed within this text:

- Movement along the x- or y-axis
- Rotation around the centroid
- Rotation around one of the vertices
- Movement along the direction of $\overrightarrow{SC}$
- Movement orthogonal to the direction of $\overrightarrow{SC}$

In the following, the new coordinates will be calculated using basic geometry following their respective movements and afterwards translated into matrix form. In order to apply translational movements a fifth coordinate must be introduced. Consequently, the coordinates of the Trilopede will have the form $P(S_x, S_y, SC_x, SC_y, 1)$.

For a movement over a distance l along the x- or y-axis it is obvious that the robot will not rotate at all and that therefore the coordinates of the vector $\overrightarrow{SC}$ will not change. The new centroid S' can be calculated as follows:

$$\overrightarrow{OS'} = \overrightarrow{OS} + l \cdot \overrightarrow{e_x} \qquad \{4\}$$

$$\overrightarrow{OS'} = \overrightarrow{OS} + l \cdot \overrightarrow{e_y} \qquad \{5\}$$

with $\overrightarrow{e_x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\overrightarrow{e_y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

From this the translation matrix T can be derived as follows:

$$T(x, l) = \begin{pmatrix} 1 & 0 & 0 & 0 & l \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \{6\}$$

For the rotation around the centroid it is clear that the coordinates of the centroid are not affected and only the rotation of the vector $\overrightarrow{SC}$ has to be applied.

$$\overrightarrow{SC'} = R(\alpha) \cdot \overrightarrow{SC} \qquad \{7\}$$

This results in the following rotation matrix R_S:

$$R_S(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \{8\}$$

For the Rotation around one of the vertices all coordinates will be changed. Therefore in first step the new direction of the vector $\overrightarrow{SC}$ must calculated. For the following calculations the vertex around which the robot is to be rotated will be called X.

The new coordinates of S and C can be calculated as follows:

$$\overrightarrow{OS'} = \overrightarrow{OX} + R(\alpha) \cdot \overrightarrow{XS} \qquad \{9\}$$

$$\overrightarrow{OC'} = \overrightarrow{OX} + R(\alpha) \cdot \overrightarrow{XC} \qquad \{10\}$$

The new vector $\overrightarrow{S'C'}$ can also be calculated as follows:

$$\overrightarrow{S'C'} = \overrightarrow{OC'} - \overrightarrow{OS'} \qquad \{11\}$$

Using {9} and {10} in {11} the following can be derived:

$$\overrightarrow{S'C'} = \overrightarrow{OX} + R(\alpha) \cdot \overrightarrow{XC} - \left(\overrightarrow{OX} + R(\alpha) \cdot \overrightarrow{XS}\right)$$

$$\overrightarrow{S'C'} = R(\alpha) \cdot \overrightarrow{XC} - R(\alpha) \cdot \overrightarrow{XS}$$

$$\overrightarrow{S'C'} = R(\alpha) \cdot \left(\overrightarrow{XC} - \overrightarrow{XS}\right)$$

$$\overrightarrow{S'C'} = R(\alpha) \cdot \left[\left(\overrightarrow{OC} - \overrightarrow{OX}\right) - \left(\overrightarrow{OS} - \overrightarrow{OX}\right)\right]$$

$$\overrightarrow{S'C'} = R(\alpha) \cdot \overrightarrow{SC} \qquad \{12\}$$

The coordinates of the three vertices can be calculated as shown in {1}, {2} and {3}. A generalised version is:

$$\overrightarrow{OX} = \overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC} \qquad \{13\}$$

$$\text{with } \beta = \begin{cases} 120° \; for \; A \\ -120° \; for \; B \\ 0° \; for \; C \end{cases}$$

Combining {9} and {13} leads to the following:

$$\overrightarrow{OS'} = \overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC} + R(\alpha) \cdot \overrightarrow{XS}$$

$$\overrightarrow{OS'} = \overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC} + R(\alpha) \cdot \left(\overrightarrow{OS} - \overrightarrow{OX}\right)$$

$$\overrightarrow{OS'} = \overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC} + R(\alpha) \cdot \left[\overrightarrow{OS} - \left(\overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC}\right)\right]$$

$$\overrightarrow{OS'} = \overrightarrow{OS} + R(\beta) \cdot \overrightarrow{SC} - R(\alpha) \cdot R(\beta) \cdot \overrightarrow{SC}$$

$$\overrightarrow{OS'} = \overrightarrow{OS} + \underbrace{\left(I - R(\alpha)\right) \cdot R(\beta)}_{\mathbf{A}} \cdot \overrightarrow{SC} \qquad \{14\}$$

Translating this into a matrix form result in the rotation matrix R_x:

$$R_X(\alpha) = \begin{pmatrix} 1 & 0 & a_{11} & a_{12} & 0 \\ 0 & 1 & a_{21} & a_{22} & 0 \\ 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \{15\}$$

with $A = \begin{pmatrix} (1-\mathrm{c}\,\alpha)\,\mathrm{c}\,\beta + \mathrm{s}\,\alpha\,\mathrm{s}\,\beta & (\mathrm{c}\,\alpha - 1)\,\mathrm{s}\,\beta + \mathrm{s}\,\alpha\,\mathrm{c}\,\beta \\ -\mathrm{s}\,\alpha\,\mathrm{c}\,\beta - (1-\mathrm{c}\,\alpha)\,\mathrm{c}\,\beta & (1-\mathrm{c}\,\alpha)\,\mathrm{c}\,\beta + \mathrm{s}\,\alpha\,\mathrm{s}\,\beta \end{pmatrix}$

using “s α c β” as abbreviation for “sin $\alpha \cdot \cos\beta$”

For movement in the direction of $\overrightarrow{SC}$, only the coordinates of the centroid will be changed. There for a translation distance l must be multiplied by the unity vector in the direction of $\overrightarrow{SC}$.

$$\overrightarrow{OS'} = \overrightarrow{OS} + l \cdot \frac{\overrightarrow{SC}}{|\overrightarrow{SC}|} = \overrightarrow{OS} + \frac{l}{l_r} \cdot \overrightarrow{SC} \quad with\ l_r = |\overrightarrow{SC}| \qquad \{16\}$$

Defining a movement in the direct of $\overrightarrow{SC}$ as forward and translating {16} into matrix form the forward matrix F can be derived.

$$F(l) = \begin{pmatrix} 1 & 0 & {}^{l}/_{l_r} & 0 & 0 \\ 0 & 1 & 0 & {}^{l}/_{l_r} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \{17\}$$

For movement orthogonal to the direction of $\overrightarrow{SC}$ the unity vector needs to be rotated.

$$\overrightarrow{OS'} = \overrightarrow{OS} + l \cdot R(\gamma) \cdot \frac{\overrightarrow{SC}}{|\overrightarrow{SC}|} = \overrightarrow{OS} + R(\gamma) \cdot \frac{l}{l_r} \cdot \overrightarrow{SC} \qquad \{18\}$$

Defining such a movement as sideward the sideward matrix S can be derived.

$$S(l) = \begin{pmatrix} 1 & 0 & {}^{l}/_{l_r} \cdot \cos\gamma & {}^{-l}/_{l_r} \cdot \sin\gamma & 0 \\ 0 & 1 & {}^{l}/_{l_r} \cdot \sin\gamma & {}^{l}/_{l_r} \cdot \cos\gamma & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \{19\}$$

with $\gamma = \begin{cases} 90° \; for\ left \\ -90° \; for\ right \end{cases}$

IV. Conclusion

This paper has described a three legged robot, the Trilopede, capable of both linear and rotational motion. The Trilopede is capable of motion without singularities and is capable of navigating over a complete 2D surface. The mathematical basis of a kinematic model for control has also been derived.

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